Forecasting the Technical Range Volatility with Moving Average (TRV-MA) model*

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November 14, 2009

Abstract

When trading, investors make decisions based on not only the security and market variances but also the technical price range.[1] However, academic literature investigating its properties is scarce. Better understandings of this risk measurement candidate are supposed to provide guidance for investors. In this paper, the statistical properties and dynamic structures of technical price range are investigated broadly with different index data, and some interesting facts are discovered:
1) the technical range volatility follows the normal distribution approximately;
2) the dynamic structure of the technical range volatility can be well described by a moving average model.
To test whether the TRV-MA model could well capture the dynamic structure of the technical price range volatility, empirical studies are performed on CAC40, DAX, FTSE, HS, NIKKEI225, S&P500, and STI. To evaluate the predicting ability of the TRV-MA model, a new criteria is proposed. Based on this new criteria, the TRV-MA model does provide sharp prediction. Interesting problems based on the technical range volatility are suggested in the conclusion, better understandings of these problems are supposed to provide better understandings of the market microstructures.

According to our knowledge, this paper is the first one to investigate the statistical properties and the dynamics of technical range volatility.

Keywords:ARCH, GARCH, TRV-MA, Price Range

1 Introduction

Volatility plays a very important role in finance, not only in asset pricing, portfolio choice but also in risk management. Estimating and modeling the volatility

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[1] The price range employed in this paper is in technical sense, different from the one adopted by Parkinson (1980).

of speculative asset prices have always been a central theme in the literature of financial economics and econometrics. As a measure of risk, volatility modeling is important to researchers who are trying to understand the nature of the dynamics of volatilities. It is also of fundamental importance to policy makers and regulators as it is closely related to the functioning and the stability of financial markets, which has direct links to the functioning and fluctuations of the real economy.

Engle (1982) developed the ARCH model and used it to estimate the means and variances of the inflation in the U. K. Bollerslev (1986) generalized the ARCH model and developed the GARCH model, which is widely used in financial engineering. As the ARCH and GARCH failed to capture the asymmetric volatility, Nelson (1989) proposed the E-GARCH model. Of course, there are many other ARCH-like models, such as the NGARCH model by Engle and Ng (1993), the TGARCH model by Zakoian (1994). For a critical review with a through survey of the ARCH literature, see Bollerslev, Chou, and Kroner (1992).

Despite the success of ARCH-family models in capturing and predicting the volatility, there are some drawbacks. Alizadeh, Brandt, and Diebold (2002), Brandt and Diebold (2006), and Chou (2005) pointed out the inaccuracy and inefficiency of ARCH-family models, because they are totally based on the closing prices of the reference period, completely ignoring the information contents inside the reference. For example, in turbulent trading days with drops and recoveries of the markets, the traditional volatility based on closing prices indicates low efficiency.

An alternative volatility proxy is price range, which in most academic literature is defined as the difference between the log highest and log lowest prices. Price range is widely investigated ever since it was first proposed by Parkinson (1980), as it is supposed to correctly show the high volatility. In this paper, the price range adopted by Parkinson is denoted as $R_{p,t}$:

$$R_{p,t} = ln(H_t) - ln(L_t) \tag{1}$$

Academic literature on the range-based volatility estimator dates back to the early 1980s.[2] Based on the assumption that the asset price follows a Geometric Brownian Motion (henceforth GBM) without drift, Parkinson (1980) proposed the following volatility estimator:

$$\widehat{\sigma}_t^2 = [ln(H_t) - ln(L_t)]^2/4ln2 \tag{2}$$

Instead of using only two point data, Garman and Klass (1980) incorporated the highest, lowest, opening and closing prices into their estimator. With the same assumption, Garman and Klass suggested the following estimator:

$$\widehat{\sigma}_t^2 = 0.5 * [ln(H_t) - ln(L_t)]^2 - (2ln2 - 1) * [ln(C_t) - ln(O_t)]^2 \tag{3}$$

In reality, it is not practical to assume no drift in financial data. In this case, neither the Parkinson nor the Garman-Klass estimators is an efficient estimator.

[2] In this paper, we denote the highest, lowest, opening and closing prices as H_t, L_t, C_t and O_t respectively.

Rogers and Satchell (1991) and Rogers, Satchell, and Yoon (1994) proposed an alternative estimator which is drift-independent, incorporating the drift term information into the highest, lowest, opening and closing prices. Their estimator is much more complex, and can be written:

$$\widehat{\sigma}_t^2 = [ln(H_t)-ln(O_t)]*[ln(H_t)-ln(C_t)]+[ln(L_t)-ln(O_t)]*[ln(L_t)-ln(C_t)] \quad (4)$$

When applied to the real data, all these three estimators are downward biased. A correction therapy was proposed by Yang and Zhang (2000).

Though elegant in form and efficient in theory, those volatility estimators suffer from their underpinnings: Geometric Brownian Motion (GBM) with or without drift.Whether or not the asset prices follow GBM is still in dispute, theoretical results based on such assumptions might be great dangers in practical applications.

When trading, investors make decisions based on not only the variances but the technical range as well. Technical range is well known in Japanese candlestick charting techniques and other technical indicators, see Nison (1991). Since the technical range is widely adopted in practical trading, and commonly used as a candidate risk measurement, and no academic literature, to our knowledge, is devoted to its statistical properties, therefore, a thorough investigation of it is quite necessary. It can be supposed that better understandings of technical range would provide good guidance for investment, risk management. In this paper, we investigate the properties of technical range for the first time, and some interesting properties are discovered.

This paper is organized as follows. In Section 2, the definition of technical range volatility is presented. The empirical statistical properties and dynamic structure of technical range volatility are examined in Section 3. In Section 4, Empirical examples using the CAC40, DAX, FTSE, HS, NIKKEI225, S&P500, and STI data to estimate the model are presented, out-of-sample forecasts and performance-evaluation are also performed. Section 5 concludes with considerations on further investigation.

2 Defining Technical Range Volatility

Different from definition of $R_{p,t}$, price range in technical analysis is defined as the difference between two extreme values: the highest and the lowest prices without taking logrithm over a fixed sampling interval.To avoid confusion, in the following passage, the price range in technical sense is denoted as R_t:

$$R_t = H_t - L_t \quad (5)$$

The technical range R_t gauges the range of the price trace, the larger is the R_t, the more volatile is the price change,or in other words, large R_t means large volatility and great risk.

For investors, one of the main questions concerned is how volatile is price change tomorrow. Is it smaller than, equal to, or larger than today's price change? Is the price change of tomorrow predictable based on historical information?

For the first question, an intuitive answer is the technical range volatility:

$$TRV_t = ln(R_t) - ln(R_{t-1}) \tag{6}$$

where TRV_t is the technical range volatility.

The meaning of (6) is obvious. If $TRV_t = 0$, one get the same price change between two consecutive time periods; if $TRV_t > 0$, the price change on time t is larger, otherwise, the price change on time t is smaller.

For the second question, the best answer is to investigate the dynamics of the technical range volatility.

The first model that provides a systematic framework for return volatility modeling is the ARCH model of Engle (1982), which is now employed in financial engineering, broadly and successfully. The basic idea of the ARCH models is based on two facts commonly observed in financial markets:
1. the shock a_t of an asset return is serially uncorrelated, but depedent;
2. the dependence of a_t can be described by a simple quadratic function of its lagged values.
Specifically, an ARCH(m) model assumes that

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t \qquad (7) \\ {\sigma_t}^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_m a_{t-m}^2 \qquad (8) \end{aligned}$$

where ε_t is a sequence of independent and indentically distributed random variables with zero mean and variance 1, $\alpha_0 > 0$, and $\alpha_i \geq 0$. ***The popularity of ARCH models lies in that they root in commonly-observed facts.***

A natural way for uncovering the commonly-observed facts is to investigate broadly, which is just how the following passage is establised.

3 Uncovering the Facts

The rule that let the data themselves tell the story is well accepted in financial studies. In this section, a broad investigation of different index data will be presented.

3.1 Data Collection

We collect the daily data of the Standard and Poors 500 (S&P500), the the Financial Times and the London Stock Exchange 100 (FTSE100), the Deutscher Aktienindex (DAX), the Cotation Assiste en Continu 40 (CAC40), the Nikkei heikin kabuka (NIKKEI225), the Hang Seng Index (HS), and the Singapore Strait Times Index (STI) for the sample period from January 3, 2000 to July 31, 2009[3]. For each day, four pieces of the price information, opening, closing,

[3] Different index with different number of observations are observed in the sample period. Observations with highest and lowest prices equal are deleted, since they are not allowed for defining the technical range volatiltiy

highest and lowest, are reported. The data set is also available from the website "www. finance.yahoo.com".

3.2 Descriptive Statistics

It is well acknowlegdged that the descriptive statistics of return volatility[4] based on closing prices are skewed, heavily-tailed and far from Gaussian distribution. It might be interesting to take a look at the descriptive statistics of the technical range volatility.

In this subsection, the descriptive statistics of both the technical range volatility and the return volatility are investiaged, and the results are presented in Table 1.

The kurtosis of the return volatility are far away from 3, with the lowest one equal to 7.110849 and the highest 10.77273, demonstrating strong evidence of being heavy tail. The Jarque-Bera Statistics of the return volatility indicate no evidence of Gaussian distribution at a significance level of 1%.

For the technical range volatility, the kurtosis are close 3, with highest equal to 2.953310, and the lowest 2.614063, indicating evidence of Gaussian distribution. The Jarque-Bera Statistics of the technical range volatility suggest some evidence of Guassian distribution. The hypothesis of Gaussian distribution can not be rejected at a significance level of 5% for the CAC40, DAX, FTSE100, and STI. For the other 3 index range volatility, the Jarque-Bera Statistics manifest that they can be better approximated by Gaussian distribution compared with the return volatility.

The Q-Q plots of each index are presented in Figure 1, 2, and the results are consistent with the descriptive statistics in Table 1. The left panel in Figure 1 and 2 are the Q-Q plots of technical range volatility versus standard normal distribution, and the right panel in Figure 1 and 2 are the Q-Q plots of return volatility versus standard normal distribution.

[Insert Figure 1, 2 about here]

3.3 Dynamic Structure of TRV_t

Time series analysis, despite its great success in engineering, physical and social sciences, the ARMA model is rarely employed to describe the dynamic structures of the financial markets. A large amount of literature has demonstrated that based on the closing prices the stock returns are highly linearly uncorrelated, and thus not linearly predicable.

In this subsection, the dynamic structure of TRV_t is explored. Following the routine steps of modeling the linear time series, we first plot in Figure 3 the (TRV_t) series for each index, and the sample autocorrelation functions (ACFs) and the sample partial autocorrelation functions (PACFs) of each index are

[4] The return volatility are denoted as r_t.

Table 1: descriptive statistics of TRV_t, and r_t.

TRV_t	CAC40	DAX	FTSE100	HS	NIKKEI225	S&P500	STI
Mean	-0.000920	-0.000580	-0.000729	-0.000266	-0.000392	-0.000558	-0.001469
Median	-0.000868	-0.005868	-0.002265	-0.009112	-0.037060	0.008838	-0.003834
Max	2.102231	1.756442	1.528031	1.693519	1.849049	1.693677	1.727605
Min	-1.473294	-1.571412	-1.568009	-1.723746	-1.435891	-1.508724	-1.517166
Std.Dev	0.494139	0.488635	0.479400	0.520317	0.519312	0.541170	0.492286
Skewness	0.040193	0.051161	0.027757	0.112211	0.214495	-0.020246	0.070082
Kurtosis	2.953310	2.796522	2.854822	2.854558	2.760211	2.614063	2.832316
Jarque-Bera	0.879318	5.249977	2.434975	6.973034	23.67008	15.10892	4.754504
Probability	0.644256	0.072441	0.295973	0.030607	0.000007	0.000524	0.092805
r_t	CAC40	DAX	FTSE100	HS	NIKKEI225	S&P500	STI
Mean	-0.000224	-9.68E-05	-0.000152	7.11E-05	-0.000258	-0.000161	1.21E-05
Median	0.000189	0.000678	0.000215	0.000339	-4.90E-05	0.000444	0.000362
Max	0.105946	0.107975	0.093842	0.134068	0.132346	0.109572	0.075305
Min	-0.094715	-0.074335	-0.092646	-0.135820	-0.121110	-0.094695	-0.092155
Std.Dev	0.015905	0.016882	0.013512	0.017163	0.016513	0.014152	0.013632
Skewness	0.035088	0.080106	-0.094542	-0.024908	-0.301668	-0.091627	-0.403164
Kurtosis	7.976503	7.110849	9.158113	10.77273	9.292219	10.62605	8.437205
Jarque-Bera	2522.463	1717.862	3825.861	5991.430	3915.691	5838.417	3022.595
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

ploted in Figure 4, and 5. Plot in the left panel is the ACFs and the PACFs in the right panel.

Surprisingly, unlike the technical range volatility the plots in Figure 3 indicates little evidence of being clustering, and appear to be quite stationary with no hints of evident heteroscedasticity.

The most surprising thing is that the all these plots in Figure 4,and 5 strongly demonstrate that TRV_t can be described by a moving average model of order q (MA(q)), as the ACFs is cut off after one lag and the PACFs decays at an exponential rate. For modeling details, Hamilton [17] wrote an excellent book on time series modeling. A moving average model of order q has the following form:

$$TRV_t = C + \varepsilon_t + \beta_1 * \varepsilon_{t-1} + ... + \beta_q * \varepsilon_{t-q} \tag{9}$$

where ε_t is the disturbance term, which is an independently, and identically distributed series, or an i.i.d series for short.

Based on closing prices, it have been long observed that the asset rate is uncorrelated, but dependent, and the dependence can be well captured by ARCH-like models. ***Based on technical range volatility, it is observed that this risk measurement has the following common facts:***

1. It follows the Gaussian distribution approximatedly;

2. It can be described by a moving average model of order q, MA(q).

[Insert Figure 3, 4, and 5 about here]

4 Forecasting Technical Range Volatility Using CAC40, DAX, FTSE, HS, NIKKEI225, S&P500, and STI

In Section 3, it has been illustrated that the technical range volatility can be captured by a moving average model of order q. In this section, model estimation, out-of-sample predictation, and model evaluation will be performed to test how well the TRV-MA model can describe the behavior of the technical range volatility.[5]

4.1 Model Estimation

Following the routines of linear time series modeling, the unit root test hypothesis is performed on each index technical range volatility. In this paper, the Augmented Dickey-Fuller (ADF) tests without trend are adopted, and the testing results are reported in Table 2. All these results in Table 2 reject the unit

[5] In this paper, the disturbance term ε_t is assumed to be Gaussian, therefore, the coefficients can be estimated straightforward, any software that is capable of linear regression can be employed.

root hypothesis at a significance level of 1%, confirming that the technical range volatility can be treated as an weakly-stationary process.

[Insert Table 2 about here]

In this paper, EViews6 is employed to help estimate the coefficients, and the results are reported in Table 3. All the coefficients estimated except for the constant term C are statistically significant at a significance level of 5%, and the adjusted R^2s indicate that the TRV-MA model fit the data series fairly good. The residual tests for each index series are performed, and the results are presented in Table 4.[6] It is easy to tell from the results in Table 4 that the linear correlation hypothesis of residuals can be rejected at a significance level of 5%,and that the is weak linear correlation of residuals-squared at a significance level of 5%, indicating that there is little evidence of heteroscedasticity, and clustering. Also, it can be observed from the Jarque-Bera Statistics that it is reasonable to assume normal distribution for the disturbance term.

[Insert Table 3, 4 about here]

4.2 Out-of-Sample Prediction and Evaluation

A volatility model must be able to forecast the volatility well; this is the central requirement in almost all financial applications(Engle, 2001). Based on the above results and (9), the out-of-sample forecasts are performed to test the predicting ability of TRV-MA(q) model. Based on the models estimated in Subsection 4.1, A one-step-forward rolling forecast with a horizon of 100 is made on each index data, and the forecasting results are plotted in Figure 6, and 7.

[Insert Figure 6, and 7 about here.[7]]

We have demonstrated in the introduction that if $TRV_t > 0$, then $ln(R_t) > ln(R_{t-1})$, indicating price range on date t is more volatile than that on date t-1; if $TRV_t > 0$, then $ln(R_t) < ln(R_{t-1})$, indicating price range on date t is less volatile than that on date t-1; otherwise, $ln(R_t) = ln(R_{t-1})$. We denote the predicted value of $TRV_t > 0$ base on information set I_{t-1} as $T\hat{R}V_t$. If TRV_t and $T\hat{R}V_t$ have the same sign, they get the same ups and downs.[8]

Based on the meaning of the TRV_t, a simple but direct way to evaluate the model predictability is to calculate the number of right ups and downs. Denote the number of right ups and downs as S, and the number of wrong ups and

[6] We denote the residual as res, the residual squared as res^2, and Ljung-Box Q Statistics as Q.

[7] The solid line represents the real value, and the dashdot line predicted value.

[8] When we say the same ups and downs, we mean that if the actual range on date t is larger (smaller)than that on date t-1 then the predicted range is also larger (smaller) than that on date t-1.

Table 2: Augmented Dickey-Fuller Test for CAC40, DAX, FTSE100, HS, NIKKEI225, S&P500, and STI.

	ADF	Critical Value(1%)	Critical Value(5%)
TRV_t of CAC40	-25.98450 (0.0000)	-3.432853	-2.862532
TRV_t of DAX	-25.82215 (0.0000)	-3.432888	-2.862547
TRV_t of FTSE100	-25.35748 (0.0000)	-3.432868	-2.862539
TRV_t of HS	-19.99704 (0.0000)	-3.433297	-2.862728
TRV_t of NIKKEI225	-19.60032 (0.0000)	-3.432954	-2.862576
TRV_t of S&P500	-26.55675 (0.0000)	-3.432881	-2.862544
TRV_t of STI	-25.72650 (0.0000)	-3.432966	-2.862582

Table 3: The regression results of CAC40, DAX, FTSE100, HS, NIKKEI225, S&P500, and STI.

RVT_t of CAC40	C	β_1	β_9			Adjusted R^2
MATR(1,9)	-0.000437 (0.5773)	-0.853459 (0.0000)	-0.046989 (0.0000)			0.416096
RVT_t of DAX	C	β_1	β_{11}	β_{16}		Adjusted R^2
MATR(1,11,16)	$-3.69E-05$ (0.9613)	-0.847885 (0.0000)	-0.027338 (0.0248)	-0.026134 (0.0316)		0.432927
RVT_t of FTSE100	C	β_1	β_9	β_{18}		Adjusted R^2
MATR(1,9,18)	$-2.25E-05$ (0.9816)	-0.835563 (0.0000)	-0.030674 (0.0111)	-0.032408 (0.0050)		0.404074
RVT_t of HS	C	β_1	β_4			Adjusted R^2
MATR(1,4)	$5.00E-05$ (0.9636)	-0.828329 (0.0000)	-0.040978 (0.0050)			0.426480
RVT_t of NIKKEI225	C	β_1	β_4			Adjusted R^2
MATR(1,4)	-0.000265 (0.7933)	-0.819765 (0.0000)	-0.061487 (0.0000)			0.397307
RVT_t of S&P500	C	β_1	β_2	β_6	β_{15}	Adjusted R^2
MATR(1,2,6,15)	$-7.32E-06$ (0.9918)	-0.894603 (0.0000)	0.057466 (0.0081)	-0.036519 (0.0044)	-0.041728 (0.0002)	0.448265
RVT_t of STI	C	β_1	β_2			Adjusted R^2
MATR(1,2)	-0.001011 (0.5105)	-0.744608 (0.0000)	-0.069940 (0.0008)			0.355967

Table 4: Residual tests for each index series.

TRV_t	CAC40	DAX	FTSE100	HS	NIKKEI225	S&P500	STI
Mean	-0.000830	-0.000870	-0.000769	-0.000152	-0.000156	-0.000518	-0.000173
Median	-0.0008214	-0.014299	-0.0011635	-0.018638	-0.015195	0.006462	-0.017173
Max	1.166113	1.535567	1.792125	1.512203	1.759898	1.848292	1.489997
Min	-1.023919	-1.076075	-1.246614	-1.123920	-1.129560	-1.469800	-1.144431
Std.Dev	0.356536	0.368626	0.369852	0.395311	0.402096	0.401936	0.395091
Skewness	0.084600	0.167720	0.153443	0.229286	0.183440	0.021339	0.217246
Kurtosis	2.793983	3.106748	3.083288	2.993970	3.002409	3.075379	2.933574
Jarque-Bera	3.974069	12.02486	9.770336	19.63026	12.63055	0.721591	18.42602
Probability	0.137101	0.002448	0.007558	0.000055	0.001808	0.697122	0.000100
$\underset{(res)}{Q(36)}$	23.305 (0.916)	42.041 (0.134)	27.960 (0.716)	26.101 (0.832)	41.973 (0.164)	38.295 (0.205)	26.926 (0.801)
$\underset{(res^2)}{Q(36)}$	30.886 (0.612)	44.349 (0.090)	51.293 (0.022)	44.568 (0.106)	50.905 (0.031)	28.367 (0.651)	28.317 (0.742)

Table 5: The $R_{s,f}$ for each index technical range volatility.

	CAC40	DAX	FTSE100	HS	NIKKEI225	S&P500	STI
$R_{s,f}$	5.25	4.26	2.70	2.85	3.76	5.67	2.13

downs as F, the ratio of S to F can be used as a criteria for model predictability evaluation.

$$R_{s,f} = S/F \tag{10}$$

where $R_{s,f}$ is the ratio. The smaller is the $R_{s,f}$, the poorer is the performance of a model:
$R_{s,f} = 0$, no predictability at all;
$R_{s,f} = \infty$, full predictability.
In this paper, the $R_{s,f}$ is used to evaluate the TRV-MA model performance.[9] The $R_{s,f}$ values for each index technical range volatility are reported in Table 5.

According to the results in Table 5, the TRV-MA model does provide very sharp predictability about range ups and downs.

5 Conclusion

In this paper, the empirical statistics and dynamic structure of the technical range volatility were investigated , based on which some interesting findings were discovered:

[9] Since this paper is mainly concerned about the technical range volatility not the variance, a comparison between the ARCH-like models and the TRV-MA(q) model is not necessary.

1) Technical range volatility can be better approximated by Gaussian distribution than return volatility can;
2) The dynamic behavior of technical range volatility is governed by moving average model.
Applications of TRV-MA model to other frequency of range intervals, say, every hour, every week, or every month will provide further understandings of the performance of the range model. Analysis using other asset prices, e.g., currency, fixed-income securities, and derivative assets, will also be useful in improving our understandings of the turbulence of the financial markets. Finally, we would like to point out an interesting problems which need further studies:
Why the coefficients β_1 are always negatively-valued? Can it be explained by neoclassical finance theory, or should we resort to the behavioral finance?
A thorough investigation of the technical range volatility would provide better understandings of the market efficiency, and would provide better guidance for investment and risk measurement.

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Figure 1: The Q-Q plots of the technical range volatility and the return volatility for CAC40, DAX, and FTSE100.

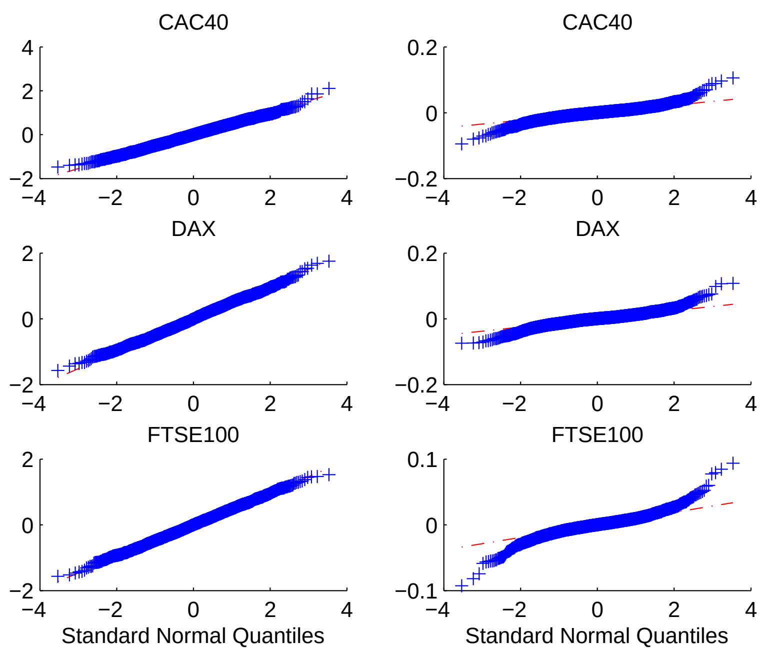

Figure 2: The Q-Q plots of the technical range volatility and the return volatility for HS, NIKKIE225, S&P500, and STI.

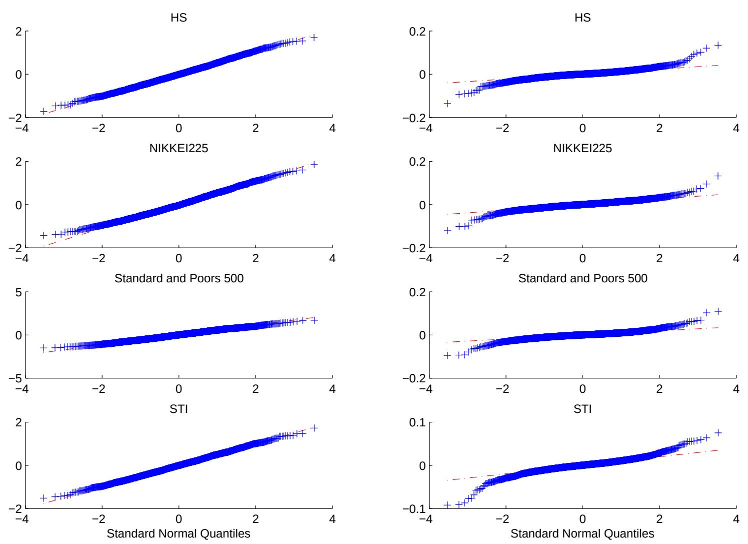

Figure 3: The TRV_t series for each index.

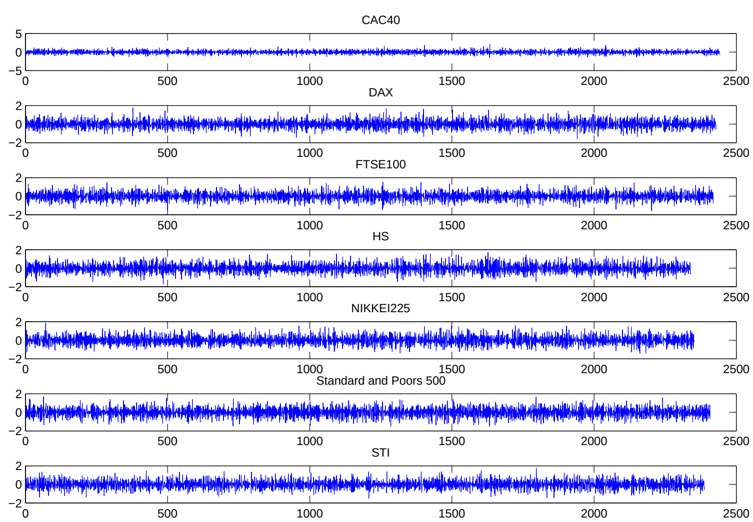

Figure 4: The ACFs and PACFs of CAC40, DAX, and FTSE100.

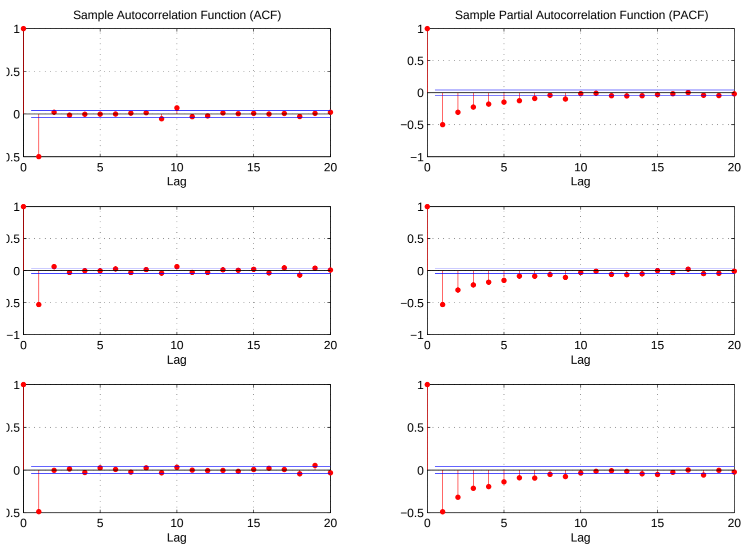

Figure 5: The ACFs and PACFs of HS, NIKKEI225, S&P500, STI.

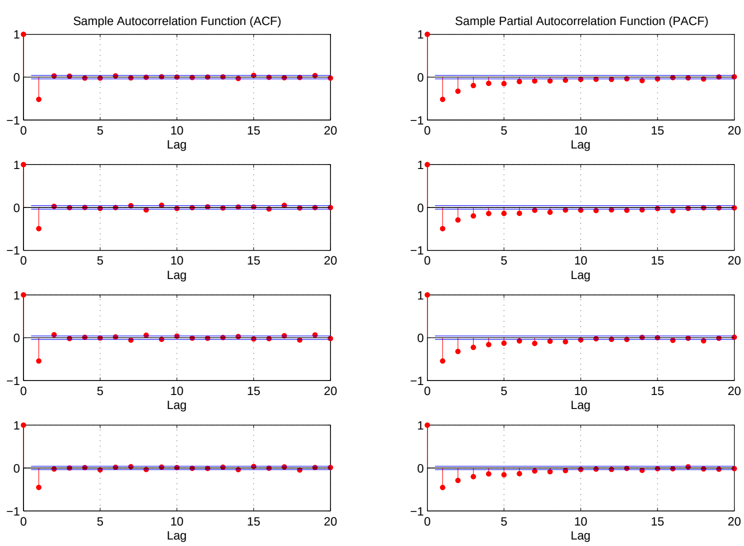

Figure 6: Forecasting results for CAC40, DAX, and FTSE

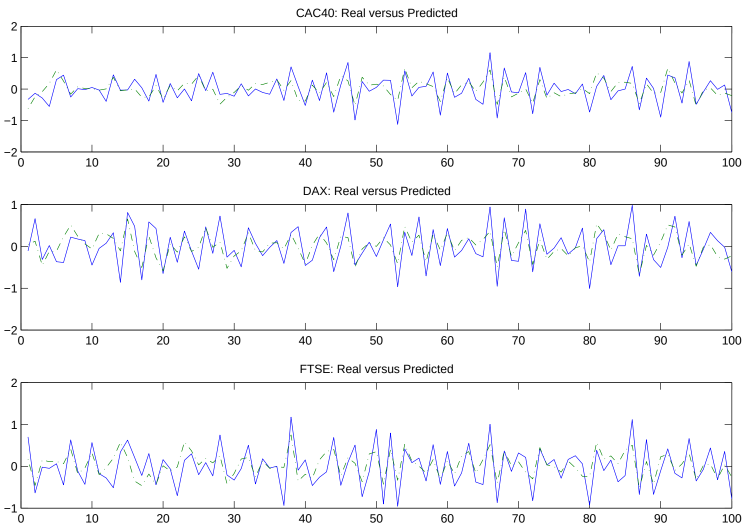

Figure 7: Forecasting results for HS, NIKKEI225, S&P500, and STI